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LETTER TO THE EDITOR

A continuous counting observation and posterior quantum dynamics

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Abstract. A stochastic model for a continuous photon counting measurement is proposed. An equation for the generating functional of the statistics of counts is found. A stochastic linear equation for the unnormalised posterior state vector of atoms, continuously collapsing without mixing, is derived by using quantum stochastic calculus methods.

The time evolution of a quantum system under continuous observation can be obtained in the framework of the quantum theory of non-demolition measurements [1–4]. In this letter we derive the corresponding posterior von Neumann and Schrödinger equations on the basis of the stochastic description of continuous non-demolition counting observation. We use the quantum stochastic counting method recently developed by Hudson and Parthasarathy for point processes in [5] and the notion of output quantum fields introduced by Gardiner and Collet [6]. The elegant Barchielli model [3, 7] of continuous measurement of the output counting process of photons emitted by a system of atoms is generalised to the case of a continuous and a mixed spectrum. Under the assumption of completeness of the non-demolition observation of the atoms, we derive the new stochastic dissipation equation, announced in [8], by which one should replace the ordinary Schrödinger equation in the case of continuous measurement, if the observed information is taken into account. As is shown in [9], equations of this type describe the continuous non-mixing collapse of the wavepacket $\varphi(t)$ whose propagation depends on measurement data up to the present instant of time $t>0$. In the case of photon emission, this collapse can be described as a sequence of electron transitions to lower atomic energy levels at random instants of photon counts.

Let us consider a quantum system 'atom + Bose (photon) field' whose unitary evolution $U(t)$ satisfies the Schrödinger quantum stochastic equation [5, 6]

$$\mathrm{d}U + KU\,\mathrm{d}t = \int (R_\varkappa\,\mathrm{d}B^\dagger(\mathrm{d}\varkappa) - R^\dagger_\varkappa\,\mathrm{d}B(\mathrm{d}\varkappa))U \qquad U(0)=\hat{I}. \tag{1}$$

Here $K=\int R^\dagger_\varkappa R_\varkappa \lambda(\mathrm{d}\varkappa)/2 + \mathrm{i}E/\hbar$, E is the energy operator of the atoms, $\{R_\varkappa, \varkappa\in\mathbb{R}^3\}$ is a family of some atom operators which define, on the right-hand side of (1), the 'energy of stochastic interaction' with the Bose field. It is described by the annihilation $B(t)$ and the creation $B^\dagger(t)$ time-dependent operator-valued measures on Borel sets $\Delta\subset\mathbb{R}^3$ of wavevectors $\varkappa=(\varkappa^i)$, $i=1,2,3$; the integral is taken over $\varkappa\in\mathbb{R}^3$ with the forward (Ito) increments

$$\mathrm{d}B^\dagger(t,\Delta) = B^\dagger(t+\mathrm{d}t,\Delta) - B^\dagger(t,\Delta) = \mathrm{d}B(t,\Delta)^\dagger.$$

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We suppose that $B(t)$ and $B^{\dagger}(t)$ represent the commutator relations in Fock space

$$[B(t', \Delta'), B^{\dagger}(t, \Delta)] = t \wedge t' \lambda(\Delta \cap \Delta') \qquad t \wedge t' = \min\{t, t'\} \tag{2}$$

with respect to a given Borel measure $\lambda(\mathrm{d}\varkappa) \geqslant 0$ on $\mathbb{R}^3$. The measure λ may be atomic: $\lambda(\mathrm{d}\varkappa) = \Sigma_k \lambda_k \delta(\varkappa_k, \mathrm{d}\varkappa)$, $\delta(\varkappa, \Delta)$ is 1, if $\varkappa \in \Delta$, and 0 if $\varkappa \notin \Delta$, in the case of a discrete wave spectrum of the quantum field, as well as dispersed, say $\lambda(\mathrm{d}\varkappa) = \mathrm{d}\varkappa := \mathrm{d}\varkappa^1\, \mathrm{d}\varkappa^2\, \mathrm{d}\varkappa^3$ in the case of a continuous spectrum.

Let us denote by $N(t)$ the time-dependent operator-valued counting measure $\Delta \mapsto N(t, \Delta)$ on $\mathbb{R}^3$, uniquely defined in the Fock space of the irreducible representation of (2) by the commutators

$$[B(t', \Delta'), N(t, \Delta)] = B(t' \wedge t, \Delta' \cap \Delta) \tag{3}$$

(the self-adjoint operators $N(t, \Delta) = N(t, \Delta)^{\dagger}$, $t > 0$, $\Delta \subset \mathbb{R}^3$ are supposed to be pairwise commutative, as well as $B(t, \Delta)$). Formally, one can consider $N(t, \Delta)$ as the integrals $\int_0^t \int_\Delta b^{\dagger}(x) b(x)\, \mathrm{d}x$ (taken over $x \in (0, t] \times \Delta$) of the Wick (normal) ordered product of generalised canonical Bose fields $b(x)$, $b^{\dagger}(x)$ on $\mathbb{R}_+ \times \mathbb{R}^3$ which define $B(t, \Delta) = \int_0^t \int_\Delta b(x)\, \mathrm{d}x$, $B^{\dagger}(t, \Delta) = \int_0^t \int_\Delta b^{\dagger}(x)\, \mathrm{d}x$ with respect to the measure $\mathrm{d}x = \mathrm{d}t\, \lambda(\mathrm{d}\varkappa)$ on $(0, t] \times \Delta$. The output [6] counting process $\hat{N}(t) = U^{\dagger}(t) N(t) U(t)$ is defined by the operator-valued measure (cf [3], equation (4.4)):

$$\hat{N}(t, \mathrm{d}\varkappa) = \lambda(\mathrm{d}\varkappa) \int_0^t \mathrm{d}s\, \hat{R}^{\dagger}_{s,\varkappa} \hat{R}_{s,\varkappa} + \int_0^t [\hat{R}^{\dagger}_{s,\varkappa}\, \mathrm{d}B(s, \mathrm{d}\varkappa) + \hat{R}_{s,\varkappa}\, \mathrm{d}B^{\dagger}(s, \mathrm{d}\varkappa)] + N(t, \mathrm{d}\varkappa) \tag{4}$$

where $\hat{R}_{t,\varkappa} = U^{\dagger}(t) R_\varkappa U(t)$. One can find it as the quantum stochastic integral $\hat{N}(t, \mathrm{d}\varkappa) = \int_0^t (\mathrm{d}\hat{N}(\mathrm{d}\varkappa)$ of the forward increments $\mathrm{d}\hat{N}(t, \Delta) = \hat{N}(t + \mathrm{d}t, \Delta) - \hat{N}(t, \Delta)$, using the quantum Ito formula [5]

$$\mathrm{d}(XY) = \mathrm{d}X\, Y + X\, \mathrm{d}Y + \mathrm{d}X\, \mathrm{d}Y \tag{5}$$

for the product $U^{\dagger} N U$ and the Hudson–Parthasarathy multiplication table

$$\begin{aligned} &\mathrm{d}B(\Delta')\, \mathrm{d}B^{\dagger}(\Delta) = \mathrm{d}t\, \lambda(\Delta' \cap \Delta) &&\mathrm{d}N(\Delta')\, \mathrm{d}N(\Delta) = \mathrm{d}N(\Delta' \cap \Delta) \\ &\mathrm{d}B(\Delta')\, \mathrm{d}N(\Delta) = \mathrm{d}B(\Delta' \cap \Delta) &&\mathrm{d}N(\Delta')\, \mathrm{d}B^{\dagger}(\Delta) = \mathrm{d}B^{\dagger}(\Delta' \cap \Delta). \end{aligned} \tag{6}$$

Note that the output process $\hat{N}(t)$ is observable due to the commutativity $[\hat{N}(t', \Delta'), \hat{N}(t, \Delta)] = 0$ for all t, t' and Δ, Δ', which means that it can be represented by a classical stochastic measure on $\mathbb{R}^3$ with values in $\{0, 1, 2, \ldots\}$. Moreover, by the property mentioned in [3, 7]

$$U(t) \hat{N}(r) = N(r) U(t) \qquad r \leqslant t$$

the output process $\hat{N}$ satisfies the non-demolition principle [4]

$$[\hat{N}(r), \hat{X}(t)] = U^{\dagger}(t)[N(r), X] U(t) = 0 \qquad r \leqslant t \tag{7}$$

with respect to any observable X of the system of atoms in the Heisenberg picture $\hat{X}(t) = U^{\dagger}(t) X U(t)$.

Let us suppose that the observed process is the quantum momentum

$$\hat{P}(t) = \int_\Delta \hbar \varkappa N(t, \mathrm{d}\varkappa) \qquad 0 \notin \Delta \subset \mathbb{R}^3$$

of the output Bose field received in a given wave region Δ with $\lambda(\Delta) = \int_\Delta \lambda(\mathrm{d}\varkappa)$ (finite wave aperture of the receiver antenna). This means that the observer counts the photons with wave momenta $\varkappa \in \Delta$ by measuring jumps of the total momentum in Δ up to the instant t. It has the value

$$\hbar\varkappa(t) = \hbar \sum_{j:\varkappa_j \in \Delta} \varkappa_j \chi(t - t_j) \qquad \chi(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geqslant 0 \end{cases}$$

for a sequence $x_j = (t_j, \varkappa_j)$, $j = 1, 2, \ldots$, definining the spectral value of the output counting measure (4) as the number $\Sigma_{j:\varkappa_j \in \Delta}\, \chi(t - t_j)$.

Let us find the equation for the generating map $\Gamma(f, t)$ of the instrument [10] describing the quantum receiver as a map $X \to \Gamma[X]$ of the algebra of atom operators X into itself defined by

$$\langle \psi | \Gamma[X] \psi \rangle = \langle \hat{X}(t)\, \hat{Y}(f, t) \rangle \tag{8}$$

where $\hat{X}(t) = U(t) X U(t)$, $\hat{Y}(f, t) = U^\dagger(t)\, Y(f, t)\, U(t)$,

$$Y(f, t) = \exp\left(\int_0^t \int_\Delta \ln f(r, \varkappa)\, \mathrm{d}N(r, \mathrm{d}\varkappa) \right) \tag{9}$$

$f(x)$ is a complex measurable function on $x = (t, \varkappa)$ with $|f(x)| \in (0, 1]$ and the mean value $\langle\, \rangle$ is taken with respect to the initial product state of the wavefunction ψ of the atom and the vacuum state vector of the input Bose field.

We find the equation by a modification of the characteristic map method described for the case of discrete spectrum Δ in [3, 7]. Taking into account that $\hat{X}\hat{Y} = U^\dagger XYU = \hat{Y}\hat{X}$ due to the non-demolition property (6) and Ito's differential rule

$$\mathrm{d}\, Y(t, f) = \int_{\varkappa \in \Delta} (f(t, \varkappa) - 1)\, Y(t, f)\, \mathrm{d}N(t, \mathrm{d}\varkappa) \tag{10}$$

for the exponential (9), one can obtain the quantum stochastic equation for the product $\hat{G}(t) = \hat{X}(t)\, \hat{Y}(t)$ by (5), (6) from (1), (10)

$$\begin{aligned} \mathrm{d}\hat{G} &= \mathrm{d}U^\dagger G U + U^\dagger\, \mathrm{d}G U + U^\dagger G\, \mathrm{d}U + \mathrm{d}U^\dagger\, \mathrm{d}G U \\ &\quad + \mathrm{d}U^\dagger G \mathrm{d}U + U^\dagger\, \mathrm{d}G\, \mathrm{d}U + \mathrm{d}U^\dagger\, \mathrm{d}G\, \mathrm{d}U \\ &= \int (f(\varkappa) \hat{R}^\dagger_\varkappa \hat{G} - G \hat{R}^\dagger_\varkappa)\, \mathrm{d}B(\mathrm{d}\varkappa) + \int (f(\varkappa) \hat{G} \hat{R}_\varkappa - \hat{R}_\varkappa \hat{G})\, \mathrm{d}B^\dagger(\mathrm{d}\varkappa) \\ &\quad + \int (f(\varkappa) - 1) \hat{G}\, \mathrm{d}N(\mathrm{d}\varkappa) + \mathrm{d}t \left(\int f(\varkappa) \hat{R}^\dagger_\varkappa \hat{G} \hat{R}_\varkappa \lambda(\mathrm{d}\varkappa) - \hat{K}^\dagger \hat{G} - \hat{G}\hat{K} \right). \end{aligned} \tag{11}$$

Here the integrals are taken over all $\varkappa \in \mathbb{R}^3$ with $f(\varkappa) = 1$ for $\varkappa \notin \Delta$. Hence, the right-hand side of the equation for the mean value (8) contains only the mean of the last differential term of (11):

$$\mathrm{d}\langle \hat{G} \rangle = \mathrm{d}t \left\langle \int f(\varkappa) \hat{R}^\dagger_\varkappa \hat{G} \hat{R}_\varkappa \lambda(\mathrm{d}\varkappa) - \hat{K}^\dagger \hat{G} - \hat{G}\hat{K} \right\rangle$$

due to the zero mean values of the other differentials with respect to the vacuum input field state.

In the Schrödinger picture $G = U\hat{G}U^\dagger = XY$, this gives the ordinary differential equation for the generating map Γ defined in (8)

$$\frac{\mathrm{d}}{\mathrm{d}t} \Gamma[X] + \Gamma[K^\dagger X + XK] = \int f(\varkappa) \Gamma[R^\dagger_\varkappa X R_\varkappa] \lambda(\mathrm{d}\varkappa). \tag{12}$$

The equation (12) in the case of the atomic measure λ was obtained by Barchielli [3] in terms of the characteristic map $\Gamma(t, \mathrm{e}^{\mathrm{i}k})$ corresponding to $f(x)=\mathrm{e}^{\mathrm{i}k(x)}$, $x=(t,\varkappa)$. The solution of this equation is given [10] by the von Neumann–Dyson series

$$\Gamma(f,t)[X]=\mathrm{e}^{-t\lambda(\Delta)}\sum_{n=0}^{\infty}\int_{0<t_1<\ldots t_n\leq t}\cdots\int V^{\dagger}(x_1,\ldots,x_n,t)\times XV(x_1,\ldots,x_n,t)\prod_{i=1}^{n}f(x_i)\,\mathrm{d}x_i \tag{13}$$

where $V(t)$ is defined by the chronological product

$$V(x_1,\ldots,x_n,t)=\mathrm{e}^{-\mathrm{i}Ht/\hbar}R_{\varkappa_n}(t_n)\ldots R_{\varkappa_1}(t_1) \tag{14}$$

of $R_{\varkappa}(t)=\mathrm{e}^{\mathrm{i}Ht/\hbar}R_{\varkappa}\,\mathrm{e}^{-\mathrm{i}Ht/\hbar}$, $\mathrm{i}H/\hbar=K-\lambda(\Delta)/2$ ($H^{\dagger}=H$ only if $R_{\varkappa}^{\dagger}R_{\varkappa}=I$ for all $\varkappa$ with $\lambda(\mathrm{d}\varkappa)\neq 0$).

Let us denote by $\omega=(x_1,\ldots,x_n)$ the chain $x_j=(t_j,\varkappa_j)$, $t_1<\ldots<t_n$, $\mathrm{d}\omega=\Pi_{j=1}^{n}\,\mathrm{d}x_j$, $\mathrm{d}x_j=\mathrm{d}t_j\,\lambda(\mathrm{d}\varkappa_j)$, and by Ω'_{Δ}, Ω'_0 the sets of all finite chains $|\omega|:=n\in\{0,1,\ldots\}$, $t_j\in(0,t]$ with $\varkappa_j\in\Delta$ and $\varkappa_j\neq\Delta$ respectively. Taking into account the fact that any chain ω is the union $\omega_0\cup\omega_1$ of the chains $\omega_0\in\Omega_0$ and $\omega_1\in\Omega_{\Delta}$ and $\mathrm{d}\omega=\mathrm{d}\omega_0\,\mathrm{d}\omega_1$, once can rewrite the series (13) in the form of the expectation

$$\Gamma(f,t)[X]=\int_{\omega\in\Omega'_{\Delta}}\Phi(\omega|t)[X]f(\omega)\,\mathrm{d}\nu(\omega|t) \tag{15}$$

of the product of $f(\omega)=\Pi_{x\in\omega}f(x)$ and

$$\Phi(\omega|t)[X]=\int_{\Omega'_0}V^{*}(\omega\cup\omega_0|t)XV(\omega\cup\omega_0|t)\,\mathrm{d}\omega_0\qquad \omega\in\Omega'_{\Delta} \tag{16}$$

with respect to the Poisson probability measure on Ω'_{Δ}

$$\mathrm{d}\nu(\omega|t)=\mathrm{e}^{-t\lambda(\Delta)}\,\mathrm{d}\omega\qquad \mathrm{d}\omega=\prod_{j=1}^{n}\mathrm{d}t_j\,\lambda(\mathrm{d}\varkappa_j). \tag{17}$$

Hence the observed quantum momentum process $\hat{P}$ up to the instant t can be described by the probability density on Ω'_{Δ} of photon counts $\omega=(x_1,\ldots,x_n)$

$$p(\omega|t)=\delta^{n}\langle\psi|\Gamma(f,t)[I]\psi\rangle/\delta f(x_1)\ldots\delta f(x_n)\Big|_{f(x)=\begin{cases}0 & \varkappa\in\Delta\\ 1 & \varkappa\notin\Delta\end{cases}}$$

with respect to $\mathrm{d}\omega$. It has a modified Poisson form

$$p(\omega|t)=\mathrm{e}^{-t\lambda(\Delta)}f(\omega|t)\qquad f(\omega|t)=\phi(\omega|t)[I] \tag{18}$$

where $\phi(\omega|t)[X]=\langle\psi|\Phi(\omega|t)[X]\psi\rangle$ is the posterior state of the atoms normalised by the probability density

$$f(\omega|t)=\int_{\Omega'_0}\|\varphi(\omega\cup\omega_0|t)\|^2\,\mathrm{d}\omega_0\qquad \varphi(\omega|t)=V(\omega|t)\psi \tag{19}$$

with respect to the Poisson measure (17). Moreover, the state $\hat{\phi}(t)$ defining the normalised posterior state

$$\hat{\rho}(\omega|t)[X]=\hat{\phi}(\omega|t)[X]/f(\omega|t)$$

almost everywhere (for $\omega : f(\omega|t) \neq 0$), satisfies (as a stochastic function) $\hat{\phi}(t) : \omega \in \Omega'_\Delta \mapsto \phi(\omega|t)$ the following stochastic equation:

$$\mathrm{d}\hat{\phi}[X] + \hat{\phi}\left(\frac{\mathrm{i}}{\hbar}(XH - H^\dagger X) - \int_{\varkappa \notin \Delta} R^\dagger_\varkappa X R_\varkappa \lambda(\mathrm{d}\varkappa)\right)\mathrm{d}t$$

$$= \int_{\varkappa \in \Delta} \hat{\phi}\left(R^\dagger_\varkappa X R_\varkappa - X\right)\mathrm{d}\hat{N}(\mathrm{d}\varkappa). \tag{20}$$

Let us prove this equation, assuming that the receiver counts the entire output field, i.e. $\lambda(\mathrm{d}\varkappa) = 0$, if $\varkappa \notin \Delta$.

In this case, since $\mathrm{d}\omega_0 = 0$ for $\omega_0 \neq \phi$, the instrument is pure $\Phi(\omega|t)[X] = V^\dagger(\omega|t) X V(\omega|t)$, i.e. the poserior state $\hat{\phi}(t)$ is defined by the stochastic vector $\hat{\varphi}(t) : \omega \mapsto \varphi(\omega|t)$, if the initial state of the atoms is described by the vector ψ. The stochastic propagator $\hat{V}(t) : (\omega, \psi) \mapsto \varphi(\omega|t)$ defined for the chain $\omega = (x_1, \ldots, x_n) \in \Omega'_\Delta$ in (14) can be written as the Wick chronologically ordered exponential function

$$\hat{V}(t) = \mathrm{e}^{-\mathrm{i}Ht/\hbar} : \overleftarrow{\exp}\left(\int_0^t \int_\Delta (R_\varkappa(r) - 1)\,\mathrm{d}\hat{N}(r, \mathrm{d}\varkappa)\right): \tag{21}$$

where $:\overleftarrow{\exp}(\;):(\omega)$ denotes the chronological product $\overleftarrow{\Pi}_{x \in \omega} R_\varkappa(t)$, $x = (t, \varkappa)$, defining the Wick ordering

$$:\overleftarrow{\exp}\int_0^t \int (R_\varkappa(r) - 1)\,\mathrm{d}\hat{N}(r, \mathrm{d}\varkappa)):(\omega) = R_{\varkappa_n}(t_n) \ldots R_{\varkappa_1}(t_1) = \overleftarrow{\prod_{x \in \omega}} R_\varkappa(t)$$

(the integral is taken over all $\varkappa \in \mathbb{R}^3$ due to the result that $N(t, \mathrm{d}\varkappa) = 0$ if $\varkappa \neq \Delta$). Hence the posterior propagator $V(t)$ satisfies the linear stochastic differential exponential equation

$$\mathrm{d}\hat{V} + \mathrm{i}H\hat{V}\,\mathrm{d}t/\hbar := \int_\Delta (R_\varkappa - I)\hat{V}\,\mathrm{d}\hat{N}(\mathrm{d}\varkappa) := \int_\Delta L_\varkappa \hat{V}\,\mathrm{d}\hat{N}(\mathrm{d}\varkappa) \tag{22}$$

obtained by differentiation of (21), where $:L_\varkappa\,\mathrm{d}N: = L_\varkappa\,\mathrm{d}N$ for $L_\varkappa = R_\varkappa - I$, because of the fact that $V(t)$ and $L_\varkappa$ are independent of $\mathrm{d}N(t) = N(t+\mathrm{d}t) - N(t) = :\mathrm{d}N(t):$. It gives the correspondence of the Wick chronological order in (21) to the Ito chronological multiplicative integral

$$\hat{V}(t) = \mathrm{e}^{-\mathrm{i}tH/\hbar} \sum_{n=0}^{\infty} \underset{0<t_1<\cdots<t_n \leq t}{\int\int_\Delta \cdots \int\int_\Delta} L_{\varkappa_n}(t_n) \ldots L_{\varkappa_1}(t_1) \prod_{j=1}^{n} \mathrm{d}\hat{N}(t_j, \mathrm{d}\varkappa_j)$$

where $L_\varkappa(t) = \mathrm{e}^{\mathrm{i}tH/\hbar} L_\varkappa \mathrm{e}^{-\mathrm{i}tH/\hbar} = R_\varkappa(t) - I$, defining the solution of (22). So the posterior state vector $\hat{\varphi}(t) = \hat{V}(t)\psi$ satisfies the stochastic wave equation

$$\mathrm{d}\hat{\varphi} + \mathrm{i}H\hat{\varphi}\,\mathrm{d}t/\hbar = \int_\Delta (R_\varkappa - I)\hat{\varphi}\,\mathrm{d}\hat{N}(\mathrm{d}\varkappa). \tag{23}$$

The linear equation (23) which corresponds, by the Ito formula (5), to the case

$$\mathrm{d}\hat{\phi}[X] - \frac{\mathrm{i}}{\hbar}\hat{\phi}[XH - H^\dagger X]\,\mathrm{d}t = \int_\Delta \hat{\phi}[R^\dagger_\varkappa X R_\varkappa - X]\,\mathrm{d}N(\mathrm{d}\varkappa)$$

of (20) for $\hat{\phi}[X] = \hat{\varphi}^\dagger X \hat{\varphi}$, can be found also from the nonlinear Schrödinger equation for the normalised state vector $\hat{\psi}(t) = \hat{\varphi}(t)/(\hat{f}(t))^{1/2}$, $\hat{f}(t) = \hat{\varphi}(t)^\dagger \hat{\varphi}(t)$, obtained in [8] with the help of the quantum filtering method.

Equation (23) explains relaxation without mixing of continuously observed atoms, emitting counting photons if $R_\varkappa$ are taken as annihilation operators $A_\varkappa$ in the energy representation $E = \Sigma_k \, \varepsilon_k |k\rangle\langle k|$ of the atoms.

References

[1] Belavkin V P 1980 *Radiotechnika i Elektronika* **25** 1445
[2] Barchielli A and Lupieri G 1985 *J. Math. Phys.* **26** 2222
[3] Barchielli A 1986 *Phys. Rev.* A **34** 1642
[4] Belavkin V P 1987 *Information Complexity and Control in Quantum Physics* ed A Blaquiere, S Diner and G Lochak (Berlin: Springer) p 311
[5] Hudson R L and Parthasarathy K R 1984 *Commun. Math. Phys.* **93** 301
[6] Gardiner C W and Collet M J 1985 *Phys. Rev.* A **31** 3761
[7] Barchielli A 1988 *Quantum Probability and Applications III* ed L Accardi and W von Waldenfels (Berlin: Springer) p 37
[8] Belavkin V P 1988 Stochastic Methods in Mathematics and Physics *Proc. 24th Karpacz Winter School in Theoretical Physics* (Singapore: World Scientific)
[9] Belavkin B P 1988 Modelling and Control of Systems in Engineering, Quantum Mechanics, Economy and Biosciences *Proc. Bellmann Continuous Workshop* (*Lecture Notes in Control and Information Sciences*) (Berlin: Springer)
[10] Davies E B 1976 *Quantum Theory of Open Systems* (London: Academic)